

INDIAN STATISTICAL INSTITUTE
Probability Theory II: B. Math (Hons.) I
Semester II, Academic Year 2017-18
Midsem Exam

Teacher: Parthanil Roy

Date: 27/02/2018

Maximum Marks: 30

Duration: 10:00 am - 12:30 pm

Note:

- Please write your name on your answer booklet.
- There are three problems each carrying 10 marks with a total of 30 marks. Solve as many as you can. Show all your works and write explanations when needed. Maximum you can score is 30 marks.
- You may use any fact proved in the class but do not forget to quote the appropriate result.
- You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc. If you are caught using any, you will get a zero grade in this exam.

1. Range of a sample is defined as the absolute difference between the minimum and the maximum sample point. For example, if the sample consists of the values 6, 8, 1, 4 and 7, then the range of the sample is 7. Let R be the range of a random sample of size n from uniform distribution on $(0, 1)$. Show that R is an absolutely continuous random variable and find its probability density function. [10]
2. Fix two positive real numbers t and λ and a nonnegative integer n . Suppose $T \sim \text{Gamma}(n + 1, \lambda)$ and $N \sim \text{Poi}(\lambda t)$. Show that $P(T > t) = P(N \leq n)$. [10]
3. Let Z_1, Z_2, \dots, Z_{20} be independent and identically distributed random variables with $Z_1 \sim N(0, 1)$. Find a joint probability density function of $U = \sum_{i=1}^{20} Z_i^2$ and $V = \frac{\sum_{i=1}^8 Z_i^2}{\sum_{i=1}^{20} Z_i^2}$. Are U and V independent? Justify your answer. [8 + 2 = 10]

Wish you all the best